AIAA JOURNAL
Vol. 37, No. 9, September 1999

Quadrupole Noise Predictions Through the Ffowcs Williams–Hawkings Equation

Sandro Ianniello*
Centro Italiano Ricerche Aerospaziali, 81043 Capua, Italy

The problem of high-speed impulsive noise prediction from hovering rotors is studied and solved in the time domain through the numerical evaluation of the nonlinear terms of the governing Ffowcs Williams–Hawkings equation (Ffowcs Williams, J. E., and Hawkings, D. L., "Sound Generation by Turbulence and Surfaces in Orbitrary Motion," *Philosophical Transactions of the Royal Society*, Vol. A264, No. 1151, 1969, pp. 321–342). Two particular resolution forms are used. The former performs the integration on a three-dimensional grid rigidly attached to the body, where the fluid velocity components are known. The latter preliminarily integrates the Lighthill stress tensor in a direction normal to the rotor plane, thus reducing the quadrupole volume integrals to some surface integrals. This last approach allows extending the calculations outside the sonic cylinder and accounts for the occurrence of multiple emission times. Both the solution forms are implemented into a retarded-time algorithm, where the integrals are determined on the present configuration of the computing grid, and into an emission-surface algorithm, where the integration is performed in the retarded domain. An analysis of the main features and limitations of the different approaches is presented, with a comparison of the corresponding numerical results and computational costs.

Introduction

DESPITE the use of different theoretical formulations and the development of sophisticated numerical tools, the evaluation of noise from rotating blades at a high transonic regime still represents a difficult problem to solve. The integral formulations based on the Ffowcs Williams–Hawkings (FW–H)[1] equation provide aeroacousticians with effective numerical tools, where the prediction of the noise signatures is achieved through a rather simple postprocessing of the kinematic and aerodynamic data. This computational simplicity dramatically disappears at a transonic or a supersonic speed, when the nonlinear terms play a significant role in the flowfield. The presence of the quadrupole three-dimensional integrals and the noncompactness of noise sources outside the sonic cylinder make the problem very complex to model numerically. Furthermore, the availability of a suitable set of aerodynamic data (involving the perturbation velocity field around the blade) is rather rare at those operating conditions. Consequently, the use of the acoustic analogy approach is generally limited to the treatment of the linear terms in the subsonic regime.

Over the last decade, the interest of aeroacousticians has progressively moved toward some alternative solution approaches. In particular, the Kirchhoff formulation[2] presently plays a leading role in the numerical prediction of high-speed impulsive (HSI) rotor noise. This method allows the achievement of a reliable prediction of acoustic waveforms through an integration on a prescribed (moving or fixed) surface, which is assumed to include all of the significant noise sources. The accuracy of the aerodynamic input and the suitable choice of the integration domain are the crucial points to achieve an accurate estimation of the acoustic pressure field. In any case, the absence of the volume integrals provides a notable reduction of the computing costs and simplifies the numerical treatment of the nonlinearities. Recently, a very interesting method has been proposed by di Francescantonio.[3] The FW–H and the Kirchhoff formulations are revised into a new, hybrid approach, exhibiting some computational advantages with respect to the usual Kirchhoff solvers; this alternative resolution form has been implemented and successfully tested in the HSI rotor noise prediction for a hovering rotor. Following di Francescantonio, Brentner and Farassat[4] have also developed a particular implementation of the FW–H equation, providing an interesting comparison between the acoustic analogy and Kirchhoff formulation for subsonically moving surfaces. The same kind of comparison has been made recently starting from supersonically rotating grids by Prieur and Rahieur,[5] who developed a very effective procedure to compute the acoustic waveforms.[6] Furthermore, the notable growth of computing power also could make competitive computational aeroacoustics methods, whose reliability in rotor noise predictions has been proved extensively.[7] In principle, these methods are very similar to the usual computational fluid dynamics methods, even though the requirement of extending the computing grid in the acoustic far field gives rise to numerical problems (dispersion and dissipation errors).

In spite of the context where the acoustic analogy seemed to be destined to play a marginal role in the numerical estimation of the HSI noise, the FW–H approach recently has been revalued. In particular, Brentner[8] has developed an efficient and robust method, successfully implemented into a retarded-time algorithm, to evaluate the subsonic quadrupole noise. Furthermore, Farassat and Brentner[9] have proposed a new integral formulation to extend the computations outside the sonic cylinder. This resolution form has been implemented into an emission-surface algorithm and has provided very encouraging results in the estimation of noise for a hovering rotor in the presence of a pronounced shock delocalization. All of the recent formulations are based on the idea of approximating the volume sources with equivalent surface sources. This simplified resolution form (known as the far-field approximation) was originally proposed by Yu et al.[10] and has been implemented by different authors.[11–14] Actually, the method may be rigorously applied only for in-plane and far-field observer positions, but this limitation is not too restrictive because the directivity of the quadrupole sources makes their contribution predominant just in the rotor plane.

The aim of the paper is to show the numerical results obtained by the different resolution forms of the FW–H equation and to compare their capabilities and limitations. The volume integration is adopted both in a retarded-time and an emission-surface algorithm and used in the HSI noise prediction for a hovering rotor at nondelocalized conditions. By using a new algorithm developed by the author and described in Ref. 15, the computations are extended in the supersonic region to check the reliability of the far-field approximation in the computation of delocalized test cases. In the following, we will refer to the new algorithm as the K-algorithm, for KURGAN, the code where it has been implemented.

Received 1 April 1998; presented as Paper 98-2377 at the AIAA/CEAS 4th Aeroacoustics Conference, Toulouse, France, 2–4 June 1998; revision received 10 December 1998; accepted for publication 1 February 1999.

*Research Engineer, Aeroacoustics Branch.

Theoretical Background

Starting with the FW–H equation, the quadrupole source term may be expressed by the following integral form:

$$4\pi p_Q(\boldsymbol{x}, t) = \frac{\partial^2}{\partial x_i \partial x_j} \int_V \left[\frac{T_{ij}}{r|1 - M_r|}\right]_{\tau^*} \mathrm{d}V \tag{1}$$

where $T_{ij} = P_{ij} + \rho u_i u_j - c_0^2 \rho \delta_{ij}$ is the Lighthill stress tensor, where ρ is the local air density, u_i the velocity of the airflow, c_0 the speed of sound in the undisturbed medium, and P_{ij} the compressive stress tensor ($P_{ij} = p\delta_{ij} - \Sigma_{ij}$, with Σ_{ij} the viscous stress tensor). The subscript τ^* indicates that for each source point all of the kernel quantities must be evaluated at the emission time, which represents, given the observer time t and location $\boldsymbol{x}$, the instant when the contribution to the noise signature was released. The double divergence appearing in Eq. (1) is changed to time derivatives, obtaining the well-known integral solution form[16]

$$4\pi p_Q(\boldsymbol{x}, t) = \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} \int_V \left[\frac{T_{rr}}{r|1 - M_r|}\right]_{\tau^*} \mathrm{d}V + \frac{1}{c_0}\frac{\partial}{\partial t} \int_V \left[\frac{3T_{rr} - T_{ii}}{r^2|1 - M_r|}\right]_{\tau^*} \mathrm{d}V + \int_V \left[\frac{3T_{rr} - T_{ii}}{r^3|1 - M_r|}\right]_{\tau^*} \mathrm{d}V \tag{2}$$

In principle, the integration domain V represents all of the space surrounding the body; from a numerical point of view, it is limited to a three-dimensional region where the presence of the moving body affects the state of the medium. The numerical solution of Eq. (2) exhibits two main problems. The first is the presence of the volume integrals, which usually require a notable computational effort. The second is the Doppler singularity appearing inside the integral kernels, which prevents the code from achieving a reliable prediction of noise when M_r (the projection of the rotational Mach number in the source-observer direction) approaches unity. Unfortunately, this condition often arises in the estimation of the HSI noise because the shock delocalization phenomena (occurring at a tip Mach number ≥ 0.88) force the inclusion of the critical supersonic region in the computations, even though the blade is rotating at subsonic tip speed. The singular behavior of the integrands may be removed through a suitable manipulation of Eq. (1), turning the actual configuration of the three-dimensional integration domain V into the retarded one. In this manner, Eq. (2) may be written in the following form:

$$4\pi p_Q(\boldsymbol{x}, t) = \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} \int_{\mathcal{V}} \left[\frac{T_{rr}}{r}\right]_{\tau^*} \mathrm{d}\mathcal{V} + \frac{1}{c_0}\frac{\partial}{\partial t} \int_{\mathcal{V}} \left[\frac{3T_{rr} - T_{ii}}{r^2}\right]_{\tau^*} \mathrm{d}\mathcal{V} + \int_{\mathcal{V}} \left[\frac{3T_{rr} - T_{ii}}{r^3}\right]_{\tau^*} \mathrm{d}\mathcal{V} \tag{3}$$

The $\mathcal{V}$ domain represents the retarded image of the computational grid (the emission or acoustic volume), where given the observer time t each source point is considered at its own emission time τ^*. From a theoretical point of view, the absence of any singularity in the integral kernels of Eq. (3) should simplify the evaluation of the quadrupole noise, despite the increased CPU time due to the presence of a time-dependent integration domain. However, the solution of Eq. (3) can be obtained only within a subsonic domain because the occurrence of multiple emission times in the supersonic region makes impracticable the reconstruction of the integration domain $\mathcal{V}$. It will be explained later how the estimation of $\mathcal{V}$ outside the sonic cylinder could be performed and, at the same time, reasons are given why such an approach is not feasible numerically.

A notable simplification of both Eqs. (2) and (3) may be achieved through the far-field approximation, which turns the quadrupole volume integrals into surface integrals through a preliminary integration along a direction normal to the rotor plane. Recently, Brentner has revised this simplified resolution form, proposing and implementing a new integral formulation (formulation Q1A, Ref. 14). By defining the quadrupole source strength on the rotor plane as

$$Q_{ij} = \int_n T_{ij}\, \mathrm{d}n \tag{4}$$

where n is the direction normal to the rotor disk, a spatial integration may be performed separately changing the quadrupole volume integrals into the following surface terms:

$$4\pi p'_Q(\boldsymbol{x}, t) = \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} \int_S \left[\frac{Q_{rr}}{r|1 - M_r|}\right]_{\tau^*} \mathrm{d}S + \frac{1}{c_0}\frac{\partial}{\partial t} \int_S \left[\frac{3Q_{rr} - Q_{ii}}{r^2|1 - M_r|}\right]_{\tau^*} \mathrm{d}S + \int_S \left[\frac{3Q_{rr} - Q_{ii}}{r^3|1 - M_r|}\right]_{\tau^*} \mathrm{d}S \tag{5}$$

The quadrupole source strength tensor Q_{ij} is nonzero over a region near the rotor blade, which extends ahead of the leading edge, behind the trailing edge, and off the blade tip. Then, the integration domain S represents a suitable two-dimensional grid, where Q_{ij} has to be known. Depending on the particular geometry of the input grid, the computation of Q_{ij} may be not simple; nevertheless, the reduction of the dimension of the quadrupole integrals represents a notable advantage and makes the computing costs for the evaluation of the nonlinear terms comparable with those required by the usual thickness and loading noise computations. Equation (5) has been implemented by Brentner in code WOPWOP+, which has provided very good HSI noise predictions for the hovering rotor at nondelocalized conditions. However, approaching an operating condition affected by a significant shock delocalization ($M_{\mathrm{tip}} \geq 0.90$), the reliability of WOPWOP+ calculations progressively decreases. In particular, the acoustic waveforms are characterized by an overestimation of the pressure disturbance and a narrower shape compared to the experimental data. These discrepancies depend on the limited grid adopted for the computations because the presence of the Doppler singularity prevents the code from including the contribution of the supersonic quadrupole sources. The WOPWOP+ calculations suggest the role played by the supersonic nonlinear terms: The signature arising from the supersonic region has to reduce the acoustic pressure peak value and to widen the resulting noise waveform. This behavior has been numerically verified by Farassat and Brentner by means of a new integral form of the FW–H equation (formulation Q2, Ref. 9).

By using the retarded configuration of the surface quadrupole grid S as the integration domain, Eq. (5) may be written

$$4\pi p'_Q(\boldsymbol{x}, t) = \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} \int_\Sigma \left[\frac{Q_{rr}}{r}\right]_{\tau^*} \mathrm{d}\Sigma + \frac{1}{c_0}\frac{\partial}{\partial t} \int_\Sigma \left[\frac{3Q_{rr} - Q_{ii}}{r^2}\right]_{\tau^*} \mathrm{d}\Sigma + \int_\Sigma \left[\frac{3Q_{rr} - Q_{ii}}{r^3}\right]_{\tau^*} \mathrm{d}\Sigma \tag{6}$$

This equation represents a singularity free expression for computing both subsonic and supersonic quadrupole sources and may be changed to formulation Q2 by introducing the time derivatives inside the integral sign. Farassat and Brentner have implemented formulation Q2 into the WOPWOP2+ code, achieving a notable improvement of the WOPWOP+ noise predictions. Nevertheless, the resulting waveforms are still affected by a fluctuating behavior, which increases with the rotational speed and the extension of the computing grid. These oscillations are probably due to an inaccurate evaluation of the Σ surface outside the sonic cylinder.

During the past two years the author's attention has been focused on the particular time evolution of the supersonic retarded domain, in the attempt to develop an effective algorithm to numerically solve Eq. (6). This algorithm has been recently implemented and tested in the numerical determination of the quadrupole source term for a nonlifting, hovering rotor blade. The very good HSI rotor noise predictions achieved by the new computational tool represent the most important result of the paper and are presented in the next section.

Numerical Results

To show the capabilities of the FW–H approach in the prediction of the HSI noise, our attention will be focused on some specific test cases, all concerning a nonlifting, hovering rotor blade. In particular, the test cases refer to a $\frac{1}{7}$-scale UH-1H rotor with straight, untwisted blades and a uniform spanwise distribution of an NACA 0012 airfoil; the rotor radius R is 1.045 m, with a blade chord of 7.62 cm. The noise signatures are computed for an observer placed in the rotor plane, at a distance of $3.09R$ from the rotor hub. Four different tip Mach numbers (0.88, 0.90, 0.925, and 0.95) are considered. The availability of the experimental data at these operating conditions[17] has made the proposed test cases a benchmark exercise for all aeroacousticians involved in HSI noise calculations.

The volume integration will be only adopted for the estimation of the subsonic quadrupole sources at the most critical condition ($M_{\text{tip}} = 0.95$). The required aerodynamic input, which consists in the three-dimensional distribution of the fluid perturbation velocity around the blade, has been obtained through an Euler code.[18] The far-field approximation will be used for all of the mentioned rotational velocities to show the reliability and the effectiveness of the K-algorithm in the assessment of the supersonic quadrupole sources. The calculations will be performed by using the Brentner's quadrupole grid,[9] and the validation of the numerical predictions will be achieved through the comparison with the experimental data.

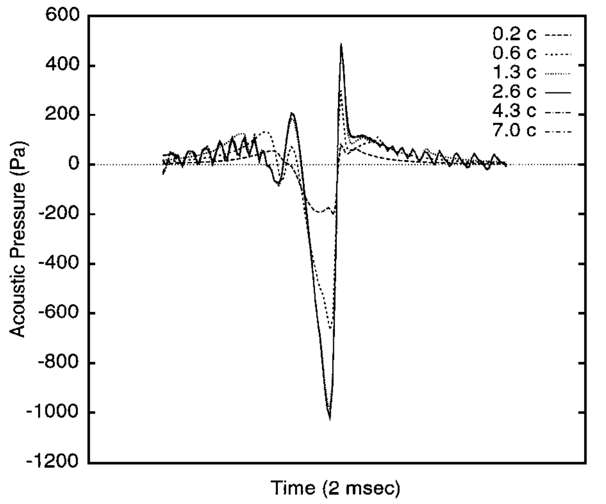

Fig. 2 Convergence of solution by moving along the direction normal to rotor plane; vertical extension of the different grids is expressed in fraction of chord *c*.

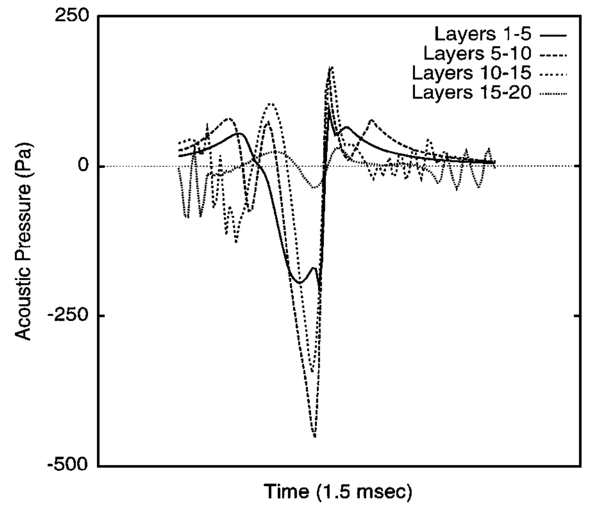

Fig. 3 Subsonic quadrupole noise contributions from subsequent sets of layers of the Euler grid.

Subsonic Volume Integration

Some good HSI rotor noise predictions obtained through the volume integration technique have been presented in the past.[13,19] Those computations have shown how a suitable choice of the three-dimensional integration domain and the use of a limited number of time steps allow achieving a good assessment of the subsonic quadrupole source term with an acceptable CPU time. Consequently, the aim is not so much to check the reliability of the numerical approach as to show its own capabilities and limitations.

The Euler aerodynamic grid accounts for 41 layers in the direction normal to the rotor disk, 97 sections along span and 129 nodes chordwise; it is characterized by a pronounced sweep and extends notably off the sonic cylinder. Nevertheless, only a subsonic region between $M_{\text{tip}} = 0.7$ and 0.99 has been considered for the computations. Figure 1 shows the comparison of the subsonic quadrupole noise predictions achieved through the numerical solutions of Eqs. (2) and (3). The integration is performed through a simple zero-order formulation, and the signatures have been determined with an azimuthal step of approximately 0.176 deg. Figure 2 shows the convergence of the acoustic calculation moving along the direction normal to the rotor plane; the signatures correspond to a different dimension of the numerical grid (expressed in fractions of the blade chord c). A converged solution is practically achieved by accounting for a mesh with a vertical extent of $2c$. The fluctuations affecting the resulting noise waveforms arise from the outer region of the grid and are imputable to the Doppler factor $(1 - M_r)$. A simple explanation of such behavior is provided by the numerical solution of Eq. (3). As explained in Ref. 15, the integration points tend to move far from the critical Doppler line, for which $(1 - M_r) = 0$; this line represents (in the rotor plane) the tangent to the sonic cylinder passing through the observer location.

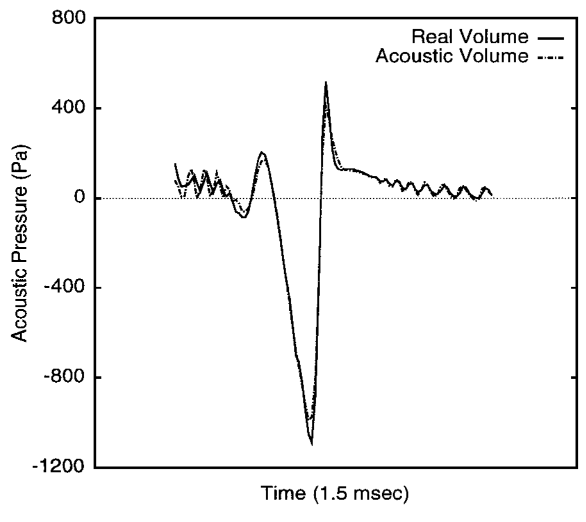

Fig. 1 Comparison of the subsonic quadrupole noise components at M_{tip} = 0.95, using Eqs. (2) and (3).

The sparseness of the source points along the Doppler line represents a dangerous feature of the region overlapping the sonic cylinder because the (fixed) number of chordwise nodes may be unsuitable to model the retarded volume and causes an inaccurate evaluation of the integration domain $\mathcal{V}$. These unaccuracies are changed to an oscillating behavior by the time derivatives of the integrals. Figure 3 shows the subsonic quadrupole noise contributions as determined from four subsequent sets of grid layers; as expected, the fluctuations arise from the outer grid regions. From a numerical point of view, the problem simply could be solved by increasing the spatial resolution of the computational domain or by using the K-algorithm to model each layer of the retarded grid; nevertheless, the requested CPU time should notably increase, and the approach should no longer be convenient.

To compare the numerical noise predictions obtained from the different solution forms, a preprocessor for the Euler data has been developed to compute the subsonic Q_{ij} distribution of Eq. (4). Figure 4 shows the very good agreement between the volume integration and the far-field approximation results for the in-plane observer location: The signature obtained by the simplified solution form does not exhibit any fluctuation and has required a very limited CPU time (26 s on a SGI Power R10000 System with respect to the 230 s required by the real volume integration code). On the other hand, moving far from the rotor plane, the reliability of the far-field approximation

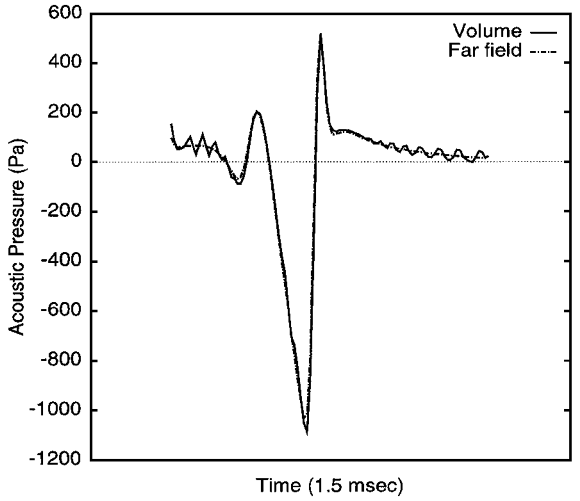

Fig. 4 Comparison between the subsonic quadrupole noise components determined through the volume integration and the far-field approximation for the hovering rotor at $M_{\text{tip}} = 0.95$.

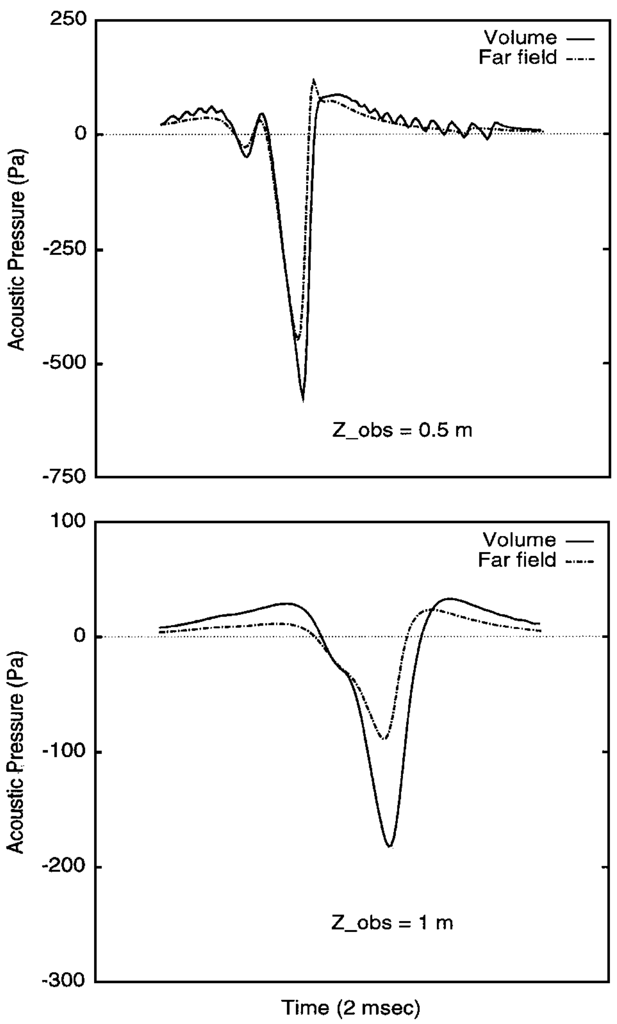

Fig. 5 Moving off the rotor plane ($Z_{\text{obs}} = 0.5$ m and 1 m at top and bottom, respectively), the agreement between the volume integration and the far-field approximation results disappears.

can be expected to decrease, and the volume integration is the only way to perform a reliable prediction of the FW–H nonlinear terms. This is confirmed in Fig. 5, where two observers placed off the rotor plane (at $Z_{\text{obs}} = 0.5$ m and $Z_{\text{obs}} = 1$ m) are considered. The subsonic quadrupole noise component determined through the far-field approximation no longer agrees with the volume integration predictions, which are assumed to be still reliable in spite of the absence of any corresponding experimental data. Ultimately, in the absence of delocalization that is in the range of rotational velocities between a tip Mach number of 0.7 and 0.88 the volume integration may be considered an effective technique in the determination of the FW–H nonlinear terms.

Supersonic Volume Integration

In principle, the numerical solution of Eq. (3) outside the sonic cylinder could be achieved by means of the K-algorithm. By determining the retarded configuration at each layer of the numerical grid, the algorithm could evaluate the three-dimensional emission volume $\mathcal{V}$ and still perform the integration with a simple zero-order formulation. From a practical point of view, this solution is unfeasible. The CPU time required to determine the retarded surface outside the sonic cylinder strongly depends on the particular test case and the resolution adopted to model the supersonic regions; in any case, it is always considerable, compared with the usual CPU time required for the estimation of the subsonic source terms. By accounting for all of the N_Z layers of the numerical grid, this time should be at least multiplied by an N_Z factor. Nevertheless, the real problem concerns the construction of the elementary volumes. In fact, given the observer time and depending on the topology of the mesh, a different distribution of the supersonic patches may occur at two subsequent layers of the grid.[15] Then to achieve an accurate reconstruction of the supersonic domain $\mathcal{V}$, the code should move between the different layers, searching for the boundaries of the three-dimensional regions. These computations represent a sort of compatibility condition between the subsequent layers at their retarded configuration and should cause an unbearable increase of the computational effort.

Far-Field Approximation

As already mentioned, the idea of reducing the quadrupole volume integrals to some equivalent surface integrals was originally proposed by Yu et al.[10] in 1978. Practically, the method assumes that all of the source points placed in a direction normal to the rotor plane have the same emission time; such an assumption allows the splitting of the three-dimensional integration of the nonlinear terms into two subsequent and simpler steps. Unfortunately, the presence of the Doppler singularity in the integral kernels and the lack of an effective numerical procedure for the determination of the supersonic Σ surface have always limited the applicability of this simplified solution form. At present, the capability of computing the supersonic acoustic surface makes the far-field approximation the most suitable way to achieve a numerical prediction of the FW–H quadrupole noise.

Figures 6–8 show the HSI noise predictions obtained for the nonlifting, hovering rotor, at four different values of the tip Mach number (0.88, 0.90, 0.925, and 0.95). The surface distributions of the Q_{ij} tensor components (as well as the experimental data) have been provided by Brentner and exactly correspond to the test cases presented in Ref. 9. Because of the notable delocalization of the shock off the blade tip, the most critical condition ($M_{\text{tip}} = 0.95$) has been treated by using all of the 38 spanwise stations of the quadrupole grid, whereas in the other test cases, the computations have been limited to 35 sections along the span. The numerical signatures refer to an azimuthal step of approximately 0.35 deg, accounting for 1024 time steps within the blade revolution period. Figure 6 shows the quadrupole noise signatures split into a subsonic and a supersonic component and determined by a suitable subdivision of the integration domain. The subsonic component includes the region of the computational grid overlapping the sonic circle. Note how the signatures' peak values are time shifted, opposing one another; this behavior explains the overestimation of the nonlinear terms usually achieved by limiting the calculations to the subsonic region. Figure 7 shows the noise components related to the different source terms of the FW–H equation and the overall noise. Note that only the $M_{\text{tip}} = 0.88$ test case includes the loading noise. In fact, the blade pressure distributions corresponding to the different tip Mach numbers were not available, and the airload at $M_{\text{tip}} = 0.88$ has been separately determined with an unsteady full-potential code, developed within the European project HELISHAPE (Task 2).[20] The reliability of such a numerical tool decreseas at higher values of rotational

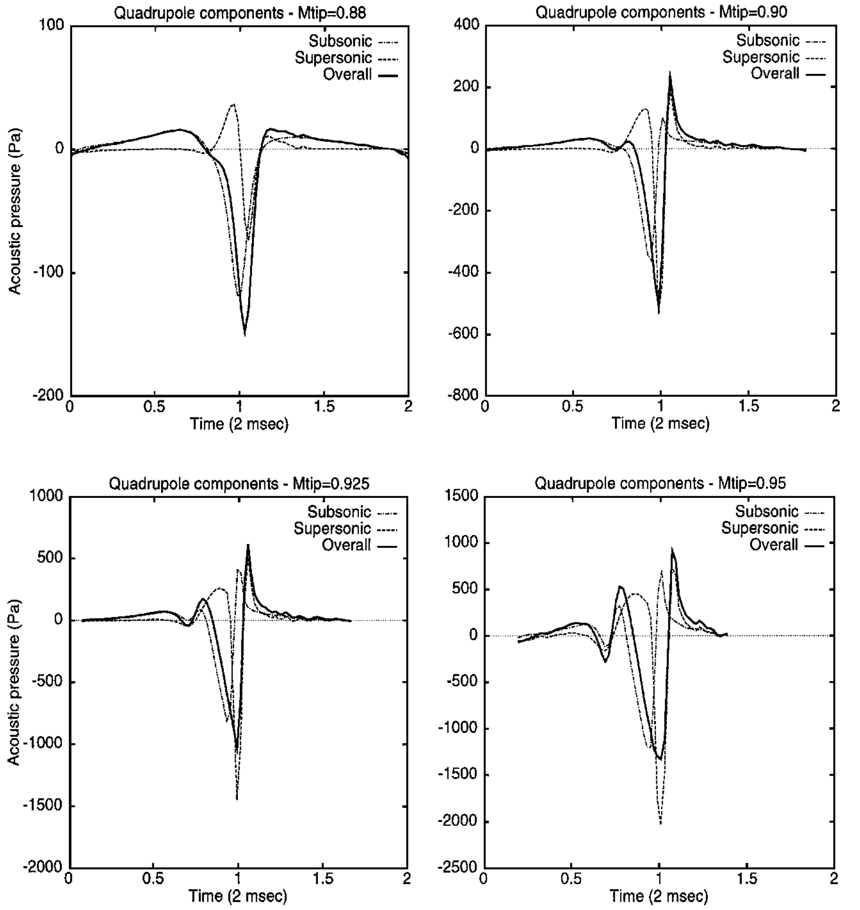

Fig. 6 Subsonic and supersonic quadrupole noise components for the hovering UH-1H rotor blade at $M_{tip} = 0.88$, 0.90, 0.925, and 0.95; subsonic components include the grid region overlapping the sonic circle.

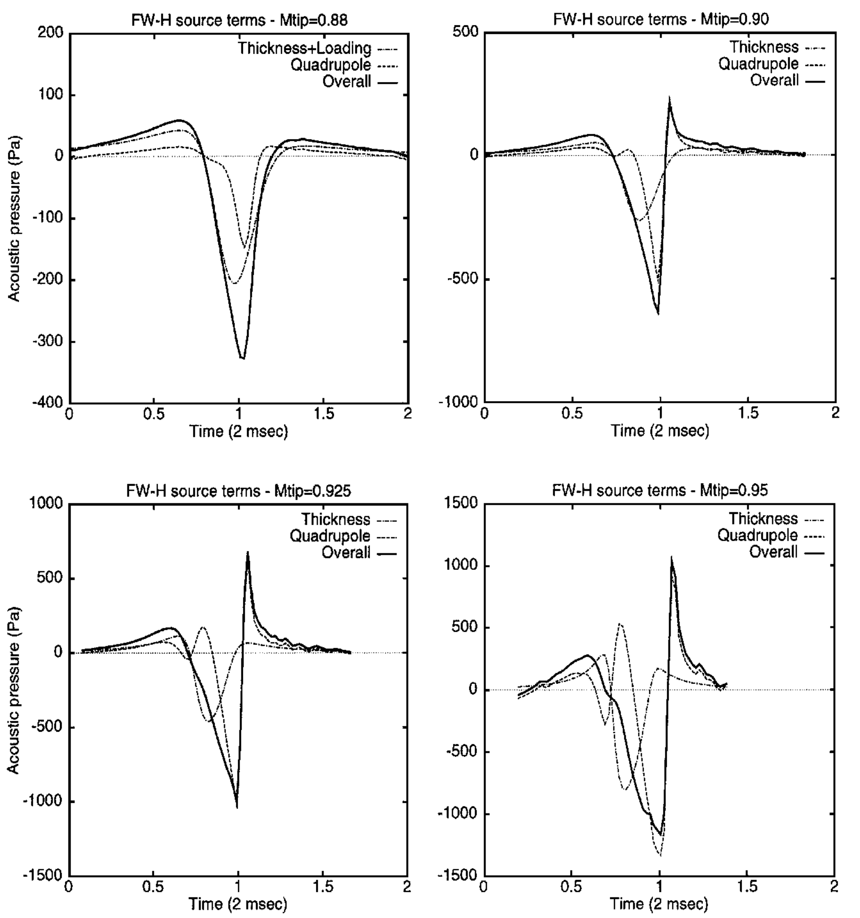

Fig. 7 Noise components related to FW–H source terms at the different tip Mach numbers; loading noise is included only at the $M_{tip} = 0.88$ test case.

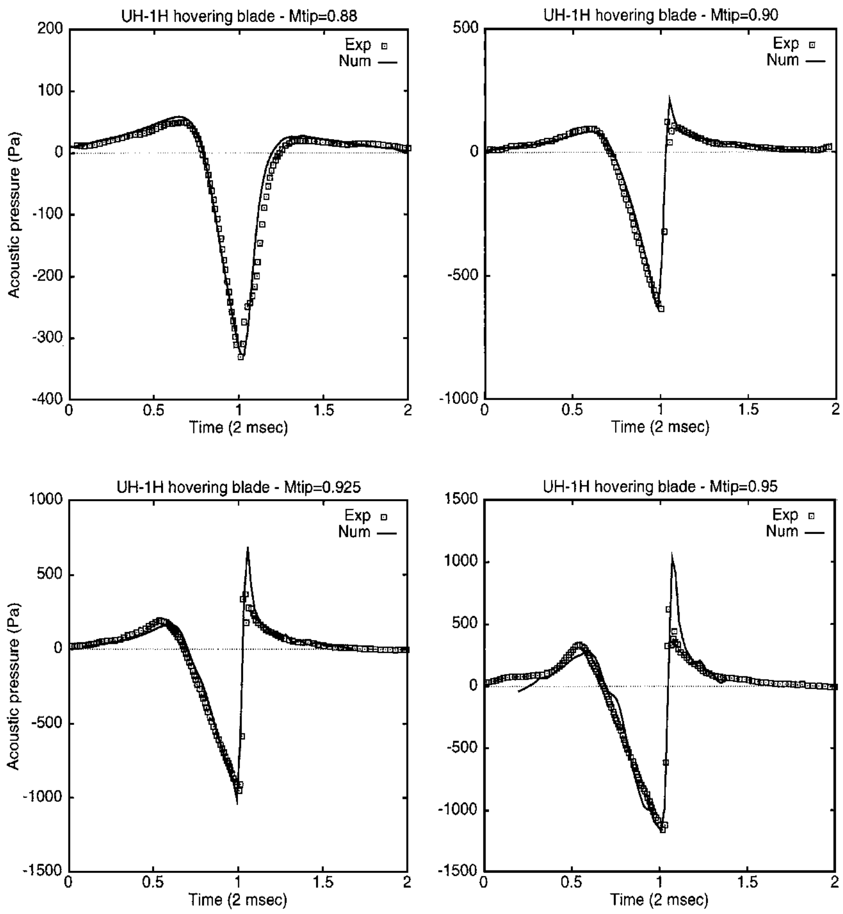

Fig. 8 Comparison between the numerical predictions achieved by the far-field approximation through the implemented K-algorithm and the corresponding experimental data.[17]

speed, so that only the thickness term has been included in the computations. On the other hand, at a $M_{\text{tip}} \geq 0.90$ the dipole contribution to the pressure disturbance becomes insignificant. At the higher rotational velocities, the out-of-phase of the acoustic pressure peak values corresponding to the linear and the nonlinear terms causes the pronounced widening of the resulting waveform. Finally, Fig. 8 shows the comparison between the computed signatures and the available experimental data.[17] The agreement is excellent. The numerical signatures exactly follow the typical asymmetrical shape of the experimental data and provide an impressively accurate prediction of the negative peak value of the acoustic pressure. Some small fluctuations still affect the supersonic quadrupole noise signature at the highest rotational speed, especially behind the characteristic recompression peak value. Nevertheless, the general smoothness of the resulting waveforms is very good and probably can be improved by using a finer resolution of the supersonic grid. The numerical results reported in the paper definitively clarify the role played by the supersonic quadrupole sources. As already verified by Farassat and Brentner, the contribution from the supersonic region widens the noise signature, increasing the asymmetrical shape of the resulting waveform. The pressure peak values corresponding to the subsonic and supersonic quadrupole components are time shifted, so that the different contributions oppose each other. This behavior has two main consequences. The first is the widening of the resulting quadrupole noise. The second is the expected reduction in the acoustic pressure peak value, compared to the numerical predictions limited to the subsonic region.

Because of the several parameters related to the numerical reconstruction of the Σ surface outside the sonic cylinder, an accurate assessment of the CPU time required for the evaluation of the supersonic quadrupole sources is very difficult. The computing cost strongly depends on the spatial resolution used to model the supersonic retarded domain, the topology of the starting grid, the tip Mach number, and in a certain sense, on the degree of smoothness one intends to achieve in the resulting noise waveform. In any case, the excellent numerical predictions reported in Figs. 6 and 8 have been obtained in a range of 1000–1500 CPU seconds on the SGI Power System (not including the time required for the evaluation of the linear terms). These values probably can be improved through an optimization of the numerical procedure, by paying specific attention to the required computing resources.

Conclusions

The aim of this paper was to show the effectiveness and the reliability of the FW–H approach in predicting HSI noise through the separate evaluation of the quadrupole source term. The volume integration technique has proved useful in a range of tip Mach numbers between 0.7 and 0.88, when the delocalization of the shock off the blade tip is not significant. At these operating conditions, the evaluation of the quadrupole volume integrals may be achieved in a limited CPU time, with no manipulation of the aerodynamic input data. Approaching higher values of the tip Mach number, volume integration becomes unsuitable. However, the far-field approximation may provide an accurate estimation of the nonlinear terms, with a very limited computing cost. The present capability of determining the supersonic retarded domain allows the use of the far-field approximation even outside the sonic cylinder. The contribution of the supersonic quadrupole sources has proved essential for an accurate estimation of the acoustic pressure field, affecting both the peak value of the signature and the resulting noise waveform.

Acknowledgments

The author wishes to thank K. Brentner from NASA Langley Research Center and M. Kuntz from DLR for kindly providing the different sets of aerodynamic data used as input for the aeroacoustic calculations presented.

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K. Kailasanath
Associate Editor